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Implications of Inflation Targeting

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Abstract

A number of countries have adopted inflation targeting in various forms as the framework for monetary policy. Even if the arguments for inflation targeting have been widely accepted, many problems of how to implement such a policy in practice remain to be solved. How shall the central bank set its operating instruments in order to control its target(s)? For how long can the actual and the targeted inflation rate be accepted to deviate under different policy regimes? How does different stabilization goals affect the variability in inflation, output and the short term interest rate? The purpose of this paper is to shed light on such questions, using a simple model for the Swedish economy.

1. Introduction

In recent years a number of industrialized countries have adopted inflation targeting in various forms as the framework for monetary policy, for example, Sweden, Finland, The United Kingdom, Spain, Israel, Australia, New Zealand, Canada and to some extent Germany and Switzerland.[1] Inflation targeting typically involves an announcement of an inflation goal, usually with some tolerance band

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[1]See Leiderman and Svensson (1995), Haldane (1995) and McCallum (1996) for detailed analyses of how different countries have implemented inflation targeting.

around the target. In practice, however, inflation targeting does not seem to rule out additional stabilization goals. A central bank can also wish to stabilize output and/or the exchange rate and it can choose to smooth the short-term interest rate.

Inflation targeting is supposed to make monetary policy more transparent to the public and easier to evaluate. Even if the arguments for inflation targeting have been widely accepted, many problems of how to implement such a policy in practice remain to be solved. How shall the central bank set its operating instruments in order to control the inflation rate? How large deviations between the actual and the targeted inflation rate can be accepted, and for how long?

As was shown in Svensson (1997a) inflation targeting may imply "inflation forecast targeting". When the central bank's only objective is the inflation rate then the inflation forecast for the shortest horizon at which the central bank can control inflation (in his theoretical model the two year inflation forecast) should equal the target. It is hence necessary for the central bank to know how long time it takes before its instrument affects inflation. In order to know how to set the interest rate so that the forecast equals the inflation rate target it is also necessary to know the values of the coefficients in the structural model of the economy.

What forecast to focus on and the form of the reaction function is also affected by the preferences of the central bank. In Svensson (1997a) it was shown that if the central bank in addition to achieving its inflation target, wants to stabilise output it should allow for a slower adjustment of the inflation forecast to the target than if the inflation target is the only goal. But it is not obvious that the central bank in practice really knows what its loss function looks like and what relative weight on the different stabilization objects it has actually chosen. This is partly because it is difficult for the central bank to foresee the actual implications of different strategies. How will the short term interest rate and output develop if the only objective for the central bank is low inflation?

In this paper it will be illustrated how the central bank's stabilization goals affect the speed with which the inflation rate adjusts toward its target after a shock and how the goals affect the determination of the short term interest rate. It will also be analysed how different policies affect the variability of different variables. By varying the objectives of the central bank it is possible to derive the "optimal policy frontier" defined in Fuhrer (1997) as "the set of efficient combinations of inflation variance and output variance attainable by policymakers".[2] For this purpose a simple quarterly macro economic model of the economy is set

[2] Another name for it, used by Taylor (1979) who was the first to do these kinds of studies, is the "optimal trade-off curve".

up together with a quarterly model of optimal monetary policy. It is assumed that the central bank in a given period chooses a sequence of current and future repo rates so as to minimize the expected sum of discounted squared future deviations of the chosen stabilization object(s) from its (their) target(s). The macro economic model is estimated on Swedish data, but it should be noticed that the purpose of the paper is not to offer a complete description of Swedish monetary policy or to make detailed policy recommendations. The macro economic model used is too simple for that purpose. It is a standard model with a single aggregate supply equation and a single aggregate demand equation. If the purpose were to describe Swedish monetary policy it would be necessary to explicitly model the impact of, for example, the exchange rate, foreign prices and foreign demand, since Sweden is a small open economy. Further it would perhaps be desirable to use an expectations augmented Phillips curve instead of the backward looking Phillips curve used here. Such extensions of the model will be dealt with in future research, where the purpose will be to analyse Swedish monetary policy in more detail.

The purpose of this paper is hence to perform an exercise in quantitative theory. It focuses on the question of how monetary policy performs under different inflation targeting regimes. It is shown that in a strict inflation targeting regime, where the central bank only has a goal for the annual inflation rate, inflation is brought back toward its target already half a year after a shock has hit the economy. This short perspective of monetary policy results in a high variability in both the short term interest rate, output and the quarterly inflation rate. With other goals in the central bank's objective function the perspective is extended quite substantially and the variability in output, the short term interest rate and in the quarterly inflation rate fall dramatically. The cost is a moderate increase in the variability of the annual inflation rate. What additional goal the central bank chooses, an output goal or an interest rate goal, is not important for these results.

As follows from above, the paper does not analyse how different specifications of the central bank's objective function affect credibility. The structural model used in this paper is such that no discretionary inflation bias due to time consistency problems appears. First of all, expectations are not explicitly modelled. There is hence no difference between a commitment solution, where the government internalizes the effects of its decisions on expectations and a discretion solution, where the government takes inflation expectations as given. But even if expectations should have been included in the model, the two solutions would have been similar since the central bank, in case it stabilizes output, is assumed to have a goal for output equal to the natural rate. Svensson (1997d) has shown that the average inflation bias then is eliminated. A state-contingent inflation bias though remains which depends on the weight on the output target. The same is true for the variability in inflation which is higher than the optimal level.

In this model inflation will hence on average be on the inflation target regardless of what weight is put on the output stabilization goal, while output on average will be on the natural output level. As shown by Svensson (1997a) both inflation and output will be mean reverting, where the weight on the output goal determines the speed. The choice of the weight on output stabilization then only effects the variability in the different variables. All solutions are hence first-best equilibria, which makes it possible to construct the "optimal policy frontier".

The remainder of this paper is organized as follows. In section 2 the structural model is presented. In section 3 a strict inflation targeting regime is analysed. Section 4 discusses the implications of additional goals. Section 6 summarizes and concludes. In the appendix the results from the econometric study are presented together with technical results.

2. The structural model

The model that is used to describe the economy is very simple. It consists of a single aggregate price equation and a single aggregate demand equation. The model looks as follows

$$\pi_{t+1} = \alpha_\pi(L)\pi_t + \alpha_y(L)y_t + \epsilon_{t+1} \tag{2.1}$$

$$y_{t+1} = \beta_y(L)y_t - \beta_r(L)(i_t - \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})) + \eta_{t+1}, \tag{2.2}$$

where $\pi_t = 4(p_t - p_{t-1})$ is the annualiced quarterly change in inflation, p_t is the (log) price level, y_t is the output gap, i_t is the interest rate, and the measure of the stance of monetary policy, ϵ_t, η_t are i.i.d. shocks, known in period t, and $\alpha_\pi(L)$, $\alpha_y(L)$, $\beta_y(L)$ and $\beta_r(L)$ are polynomial lag operators. The annual change in inflation is, $\pi_t^y = \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) = p_t - p_{t-4}$, that is the four-quarter change in inflation.

The first equation specifies a Phillips curve relationship. The lagged variables in the inflation equation are included to capture inflation expectations, assuming that people form their expectations based on recent observations. They also capture the stickiness of inflation due to staggered pricing and overlapping contracts. It is assumed that the coefficients on lagged inflation sum to one. This is consistent with the Phillips curve being vertical in the long run; in the long run there is hence no trade off between inflation and output.[3] The second term is the deviation of output from its natural rate. Equation (2.2) specifies output as a function of lagged output, which captures the persistence in output and the lagged real interest rate.

Estimating the model above using Swedish quarterly data resulted in the following two equations for the quarterly inflation rate and for the quarterly output gap:[4]

$$\begin{aligned} \pi_{t+1} &= 0.34\pi_t + 0.18\pi_{t-1} + 0.36\pi_{t-2} - 0.28\pi_{t-3} + 0.4\widetilde{\pi}_{t-4} + 0.3y_t \quad (2.3) \\ &\quad + \widehat{\epsilon}_{t+1}, \\ y_{t+1} &= 0.9y_t - 0.13(i_t - \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})) + \widehat{\eta}_{t+1}, \quad (2.4) \end{aligned}$$

where $\widetilde{\pi}_t = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3} + \pi_{t-4} + \pi_{t-5})/6$, π_t is the quarterly change in inflation (annual rate of the CPI, measured as the deviation from a constant mean)[5], y_t is the output gap[6], i_t is the overnight interest rate (annual rate, measured as the deviation from a constant mean)[7]. In appendix A a more thorough description of the empirical results are presented. For later purposes the system is rewritten in matrix notation as

$$X_{t+1} = AX_t + Bi_t + \epsilon_{X,t+1}, \quad (2.5)$$

where $X_t \equiv (\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, \pi_{t-5}, \pi_{t-6}, \pi_{t-7}, \pi_{t-8}, \pi_{t-9}, y_t, i_{t-1}, i_{t-2})'$ is

[3]If some other nominal variable is included among the explanatory variables, for example the money suply or foreign prices, then the coefficient on these variables and the own lags should sum to one. In this case the Phillips curve is also vertical in the long run and the model is homogenous.

[4]The output equation is estimated on quarterly data from the end of 1985 to 1996. The reason for not using earlier data is that the Swedish financial markets were highly regulated until 1985. Monetary policy was before 1985 conducted by changes in the regulation of banks and other financial institutions. Since then the instrument of monetary policy has been the short term interest rate. The inflation equation on the other hand is estimated using data from 1970.

[5]The inflation rate is seasonally adjusted.

[6]Calculated with a HP-filter (λ=100000), using the log of GDP, seasonally adjusted.

[7]The overnight interest rate is used instead of the marginal interest rate and the repo rate. It is then possible to run the regressions from the end of 1985. If instead the marginal interest rate had been used, the start date would have been the first quarter of 1987.

a (13×1) vector of the thirteen predetermined variables at time t,[8] and

$$\underset{(13\times13)}{A} = \begin{bmatrix} \sum_{j=1}^{4}\alpha_j e_j + \sum_{j=5}^{10}\frac{\alpha_5}{6}e_j + \alpha_6 e_{11} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ \frac{\beta_2}{4}e_{1:4} + \beta_1 e_{11} \\ e_0 \\ e_{12} \end{bmatrix} \tag{2.6}$$

where e_j , j=1,2,...12 is a (1×13) row vector, with element j equal to unity and all other elements equal to zero, e_0 is a (1×13) row vector with all elements equal to zero and where $e_{1:4}$ is a (1×13) row vector, with elements 1 to 4 equal to unity and all other elements equal to zero.

$$\underset{(1\times13)}{B'} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 1 & 0 \end{bmatrix} \tag{2.7}$$

$$\underset{(1\times13)}{\epsilon'_X} = \begin{bmatrix} \epsilon_\pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \epsilon_y & 0 & 0 \end{bmatrix}. \tag{2.8}$$

As can be seen the quarterly inflation rate is increasing in four lags of it its own, an average of lag five to lag ten and lagged output. Output is increasing in lagged output and decreasing in the short term interest rate lagged one period. This quarterly model hence suggests that monetary policy, measured by the short term interest rate, starts to affect output after one quarter and prices after two quarters, with a maximal effect on prices after around one and a half years.[9]

When first estimating the inflation equation the coefficient on the own lags summed to 0.91. It was hence reasonable to assume that the inflation process,

[8] Two interest rates are included to be able to calculate the standard deviation for the interest rate difference.

[9] If the system of the two estimated equations for the quarterly inflation rate and the output gap is hit by a one-standard deviation shock in the nominal interest rate the maximal impact on the annual inflation rate is after around one and a half year. The accumulated effect on the annual inflation rate of course continues to rise but now at a slower speed.

using data from this period, was characterized by a unit root. In favour of this assumption is the result from an Augmented Dickey-Fuller test which showed that a unit root in the quarterly inflation rate could not be rejected. Adding an exogenous variable as the output gap which is stationary - on a 5 % significance level the hypothesis that the output gap contains an unit root was rejected - should then not change the result. The constraint that the coefficients summed to one was therefore imposed and the inflation equation was estimated in the following standard form

$$\Delta\pi_{t+1} = \alpha'_{\pi}(L)\Delta\pi_t + \alpha_y(L)y_t + \epsilon_{t+1}, \tag{2.9}$$

where $\alpha'_{\pi,i} = -\sum_{j=i+1} \alpha_{\pi,j}$ for $i = 1, 2, 3, ...$ and for $j = 1, 2, 3, ...$ and $\alpha'_{\pi}(L) = \alpha'_{\pi,1}L + \alpha'_{\pi,2}L^2 + \alpha'_{\pi,3}L^3 + ..$, and $\alpha_{\pi}(L) = \alpha_{\pi,1}L + \alpha_{\pi,2}L^2 + \alpha_{\pi,3}L^3 + ...$ From this equation it was then possible to derive equation (2.3) above.

The characteristics of this structural model are illustrated in figure 1. There the dynamic responses of the variables to different shocks are shown. This case is called the baseline-case. Here monetary policy is passive, the short term interest rate is not adjusted in response to shocks.

First it is shown how inflation and output respond to a single one-standard deviation shock to supply. If a supply shock, which here is equal to a shock in inflation, hits the economy and monetary policy is passive there is no mechanism to bring inflation back to its initial level. As a start the inflation rate rises as a response to the price shock. As a consequence the real interest rate falls, since the nominal interest rate is unchanged, and output increases. Inflation then continues to rise. The result follows from the fact that there is a unit root in the inflation process; with a unit root, there is no mechanism to bring inflation back to its original level once inflation moves away from it.

If an output shock (a demand shock) hits the economy both inflation and output will start to rise and continue to do that until some negative shock hits the economy.

3. A strict inflation targeting regime

To this simple macro economic model, a model of optimal monetary policy is added. When modelling the central bank's behaviour it is as a start assumed that the central bank's only goal is to stabilize the annual inflation rate. The inflation target is π^*. It is hence assumed that the central bank uses a loss function in period t equal to

$$L(X_t) = \frac{1}{2}\left[(\pi_t^y - \pi^*)^2\right]. \tag{3.1}$$

The instrument of the central bank is the short term interest rate. Formally the objective for the central bank in period t is then to choose a sequence of current and future repo rates so as to minimize its intertemporal loss function,

$$J(X_t) \equiv \min_{\{i_\tau\}_{\tau=t}^{\infty}} \mathrm{E}_t \sum_{\tau=t}^{\infty} \delta^{\tau-t}\frac{1}{2}(\pi_\tau^y - \pi^*)^2, \tag{3.2}$$

subject to equation (2.3) and (2.4), where $0 < \delta < 1$ is a discount factor. The central bank then minimizes the expected sum of discounted squared future deviations of the four quarter inflation rate from the annual inflation target. When solving this optimization problem it is possible to rewrite it as a sequence of one-period problems as shown in Svensson (1996). The central bank then finds the optimal repo rate in year t as the solution to the one-period problem

$$\min_{i_t} \frac{1}{2}\mathrm{E}_t\delta^2(\pi_{t+2}^y - \pi^*)^2. \tag{3.3}$$

The first-order condition is

$$\mathrm{E}_t\left[\delta^2(\pi_{t+2}^y - \pi^*)\right]\frac{\partial \pi_{t+2}^y}{\partial i_t} = 0, \tag{3.4}$$

or

$$\pi_{t+2|t}^y(i_t) - \pi^* = 0, \tag{3.5}$$

where $\pi_{t+2|t}^y(i_t)$ is defined as

$$\pi_{t+2|t}^y(i_t) \equiv \mathrm{E}_t\left[\pi_{t+2}^y \mid X_t, i_t\right]. \tag{3.6}$$

This is the inflation forecast made in period t conditional upon information available in period t, X_t, and conditional on the repo rate set this period, i_t. The repo rate in quarter t should hence be set so that the forecast of the four quarter inflation rate quarter $t+2$, conditional upon information available in period t, equals the annual inflation target. The forecast for quarter $t+2$ is the first inflation forecast that the central bank can control. The perspective of the central bank is hence very short if the structural model is assumed to be given by equation (2.3) and (2.4) and if the central bank is assumed to only have a goal for the inflation rate.

To derive the reaction function for the interest rate from this result the FOC has to rewritten. The annual inflation rate in period $t+2$ is given by

$$\pi^y_{t+2} = \frac{1}{4}\left(\pi_{t+2} + \pi_{t+1} + \pi_t + \pi_{t-1}\right) \tag{3.7}$$

or in matrix notations

$$\pi^y_{t+2} = e_{1:4} X_{t+2} \tag{3.8}$$

where

$$\underset{(1\times 13)}{e_{1:4}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.9}$$

Further, from equation 2.5 it follows that

$$\begin{aligned} X_{t+2} &= AX_{t+1} + Bi_{t+1} + \epsilon_{X,t+2} \\ &= A\left(AX_t + Bi_t + \epsilon_{X,t+1}\right) + Bi_{t+1} + \epsilon_{X,t+2} \\ &= AAX_t + ABi_t + A\epsilon_{X,t+1} + Bi_{t+1} + \epsilon_{X,t+2}. \end{aligned} \tag{3.10}$$

Expressed in terms of known variables in period t, the annual inflation forecast two quarters ahead can hence be written as

$$\begin{aligned} \pi^y_{t+2|t}(i_t) &= e_{1:4} X_{t+2|t} \\ &= e_{1:4} A\left(AX_t + Bi_t\right) \end{aligned} \tag{3.11}$$

It depends on the repo rate set today and the present observations on the state vector X_t, that is, the quarterly inflation rate today and ten quarters back. The FOC can hence be rewritten in the following form

$$e_{1:4} X_{t+2|t} = e_{1:4} A\left(AX_t + Bi_t\right) = 0 \tag{3.12}$$

assuming the target is equal to zero. The reaction function for the interest rate is then given by

$$i_t = -\left(e_{1:4}AB\right)^{-1} e_{1:4} A^2 X_t \tag{3.13}$$

The reaction function is hence written as a function of the state vector as of today. It follows that output, besides the quarterly inflation, is important for how the short term interest rate is set today. The reason that today's output is included in the reaction function is of course because it helps predicting future inflation. If the central bank uses this reaction function the annual inflation rate forecast two quarters ahead will always equal the target. How the interest rate shall be adjusted in case the forecast is not on the target can easily be shown. Equation (3.11) can be rewritten as

$$\begin{aligned}\pi^y_{t+2|t}(i_t) &= e_{1:4}A\left(AX_t + Bi_{t-1} + B\Delta i_t\right) \\ &= \pi^y_{t+2|t}(i_{t-1}) + e_{1:4}AB\Delta i_t, \end{aligned} \tag{3.14}$$

where we have used (3.11).

From the first order condition and this expression for the forecast of the four quarter inflation rate it follows that

$$\Delta i_t = D\left(\pi^y_{t+2|t}(i_{t-1}) - \pi^*\right). \tag{3.15}$$

where

$$D = -\frac{1}{e_{1:4}AB} > 0 \tag{3.16}$$

If the inflation forecast before the repo rate in this period is set, is above the target then the interest rate should be increased, if the forecast is below the target the interest rate should be decreased.

Equilibrium inflation in period $t+2$ is then

$$\pi^y_{t+2} = \pi^* + e_{1:4}A\epsilon_{X,t+1} + e_{1:4}\epsilon_{X,t+2} \tag{3.17}$$

It will deviate from the target with disturbances that occur after the repo rate is set, that is the disturbances that occur within the control lag.

Using the estimated values of the coefficients, the reaction function is

$$\begin{aligned} i_t &= -102.6\pi^* + 42.2\pi_t + 41.3\pi_{t-1} + 5.4\pi_{t-2} - 9.4\pi_{t-3} \\ &\quad +10.3\widetilde{\pi}_{t-3} + 13.7\widetilde{\pi}_{t-4} + 17.2y_t, \end{aligned} \tag{3.18}$$

the interest rate will be adjusted according to

$$\Delta i_t = 102.6\left(\pi^y_{t+2|t,i_{t-1}} - \pi^*\right), \tag{3.19}$$

and the equilibrium inflation is

$$\pi^y_{t+2} = \pi^* + 0.335\epsilon_{t+1} + 0.075\eta_{t+1} + 0.25\epsilon_{t+2}. \tag{3.20}$$

The coefficients in the reaction function are hence very large and the interest rate will change dramatically in response to deviations in the inflation forecast from the inflation target.[10] The reason and consequences of this will now be analysed in the next section.

[10]It should be noted that the reaction function derived is such that the interest rate can become negative, which is a problem.

3.1. Responses to shocks

To analyse the implication of a strict inflation targeting regime the dynamic responses of the variables to different shocks will be analysed, that is, it will be illustrated how inflation, output and the interest rate respond to a single one-standard deviation shock to a demand or supply. Then the unconditional variances for the different variables will be derived for the case where the central bank only targets the inflation rate. For simplicity the target is set equal to zero.

Figure 2 shows the dynamic responses after a demand shock. A central bank that only targets the annual inflation rate, raises the nominal and the real interest rate substantially. This creates a massive output-gap which depresses inflation. After only half a year the annual inflation rate is back on its target, which is in line with the results derived above. The interest rate and output fluctuate heavily. A consequence of a strict inflation targeting regime is hence a high instability in the interest rate, output and also in the quarterly inflation rate regardless of what shock hits the economy. One reason for this instrument instability is that policy is set only for one period at a time. When the central bank only cares about inflation, the interest rate today, in this model, is determined only by the first controllable inflation forecast which is the annual inflation forecast two quarters ahead. The central bank does not take into account how the interest rate today affects the annual inflation forecast three quarters ahead and forward since that is not necessary. Instead the interest rate next period is set so that the annual inflation forecast three quarters ahead is in line with the target and so on. The central bank does not take into account that setting the interest rate so that the first controllable inflation forecast equals the target means that the interest rate and therefore output has to vary a lot since the initial effects of a change in the instrument are relatively small.

Assume for example that there is a one-standard deviation shock to the quarterly inflation rate (at an annual rate) in period $t = 0$. The quarterly inflation rate then rises with the shock, 1 percent, and the annual inflation rate increases to 0.25 percent, ($\pi^y_{t+0} = \frac{1}{4}(\pi_{t+0} + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}) = 0.25)$). The forecast of the next period's quarterly inflation rate is hence 0.34 percent, since the only known thing affecting the quarterly inflation rate in period $t+1$ is the quarterly inflation rate the quarter before, (the coefficient on the first lag in the inflation equation is 0.34). The forecast of the annual inflation rate in that period is $\pi^y_{t+1|t} = \frac{1}{4}(0.34 + 1 + 0 + 0) = 0.335$ percent. The following quarter monetary policy starts to affect inflation. Since it is the annual inflation rate that is targeted and since the target is assumed to be zero, the nominal and real interest rate in period t have to be increased, to depress output in $t+1$, so that the quarterly inflation rate in $t+2$ is predicted to fall with around 1.34 percent. The annual inflation rate in period $t+2$ will then return to target, $\pi^y_{t+2} = \frac{1}{4}(-1.34 + 0.34 + 1 + 0) = 0$. Since the effect of a rise in the interest rate in period t on the inflation rate in period $t+2$ is small the rise in the interest rate has to be huge.

Given the persistence in output, also output in period $t+2$ will be low, with unchanged policy and given that no other shocks hit the economy. Because of the persistence in output and also in the quarterly inflation rate, the annual inflation rate three quarters ahead falls below the target. The interest rate in quarter $t+1$ hence has to be decreased below its trend for the annual inflation rate in $t+3$ to hit the target, and again since the impact is small after only two quarters the fall has to be of a substantial size. These fluctuations around the trend levels will continue until the annual inflation forecast stabilizes around the target for unchanged monetary policy.

As can be seen in figure 2 it is not only the interest rate and output that fluctuate heavily but also the quarterly inflation rate. This is because the quarterly inflation rate has to be adjusted so that the annual inflation rate hits its target. As analysed above the quarterly inflation rate in period t was 1 percent, in period $t+1$ it was 0.34 percent and in $t+2$ it was -1.34 percent. In $t+3$ it thus has to be almost 0 percent for the annual inflation rate to hit the target and in $t+4$ it has to be almost 1 percent. These swings will only slowly be reduced.

The high instability in the instrument is not only a result of the way policy is set and the persistence in output and inflation, but of course also a result of the way the economy is structured. If monetary policy had a larger impact on output or if output had a larger impact on inflation, then the variability in the interest rate would be reduced. The same is true for a longer monetary policy transmission mechanism, as long as the inflation process is not characterised by a unit root. If the transmission mechanism worked slower than in this model and if the own lags summed to less then one, then the initial increase in the interest rate would not have to be so large after for example a supply shock. This is because the annual inflation rate by itself would move back towards its trend level. The longer the transmission mechanism is, the closer would the annual inflation rate be to the target the period that monetary policy started to affect inflation. The fluctuations in the interest rate, output and also in the quarterly inflation rate would hence be smaller. With a unit root in the inflation equation the opposite is true, monetary policy then has to react aggressively on shocks that hit the economy.

In table 3.1 the calculated standard deviations are summarized for all variables.

Table 3.1: Theoretical unconditional standard deviations.

Quart. infl.	Annual infl.	Output	Int. rate	Int. rate diff.
66.73	1.27	341	3586	5195

The conclusion from the exercise made above is hence that the outcomes from a strict inflation targeting regime are quite extreme. The perspective of monetary policy is then very short, only half a year. This is probably not the perspective that central banks usually have. A perspective of one to two years is probably what is more common. From the econometric study made here it was suggested that this is when monetary policy has its largest impact on inflation. Further the coefficients in the reaction function for the short term interest rate were substantial, e.g. far from the weights suggested by Taylor (1993); this will be analysed further below. Finally the variability in the short term interest rate and output derived under an inflation targeting regime are probably not the kind of outcomes that any central banker would find desirable.

One explanation for the discrepancy between the theoretical outcomes and the outcomes that one can actually observe in the real world can be that the loss function used above is incorrect, e.g. because central banks do care about other things than price stability, such as output and/or interest rate stability. In the next section the model of optimal monetary policy will therefore be rewritten with additional goals in the loss function. Another possible explanation is that the structural model is incorrect. This is discussed in section 5.

4. Additional goals

Solutions will now be derived for the case when the central banks loss function includes additional goals. It is assumed that the central bank besides targeting the annual inflation rate also targets output and/or the interest rate. It is assumed that the central bank can choose to have a target for output equal to the natural rate, or expressed differently, it can choose to minimize the output gap, y_t. The relative weight the central bank puts on the output goal is λ. The central bank can also have preferences concerning the interest rate. First it can be interested in smoothing the quarterly changes in the interest rate, $(i_t - i_{t-1})$. Second it can have a target for the interest rate level equal to the interest rate's long- run mean, here normalised to zero. The relative weights on the interest rate goals are μ and

ν. The central banks loss function in period t can then be written as

$$L(X_t) = \frac{1}{2}\left[(\pi_t^y)^2 + \lambda y_t^2 + \mu(i_t - i_{t-1})^2 + \nu {i_t}^2\right]. \tag{4.1}$$

The objective for the central bank in period t is then to choose a sequence of current and future repo rates so as to minimize its intertemporal loss function

$$J(X_t) \equiv \min_{\{i_t\}_1^\infty} \mathrm{E}_t \sum_{t=1}^{\infty} \delta^{t-1} \frac{1}{2}\left[(\pi_t^y)^2 + \lambda y_t^2 + \mu(i_{t-1} - i_t)^2 + \nu {i_t}^2\right], \tag{4.2}$$

subject to equation (2.3) and (2.4). The central bank then minimizes the expected sum of discounted squared future deviations of the four quarter inflation rate from the annual inflation target, the expected sum of discounted squared future output gaps, and the expected sum of discounted squared future variations in the interest rate. Since the optimization problem now consists of more state variables it is appropriate to rewrite the problem in matrix notation and use existing standard solutions. In matrix notation the optimization problem can be written as

$$J(X_t) \equiv \min_{\{i_t\}_1^\infty} \mathrm{E}_t \sum_{t=1}^{\infty} \delta^{t-1} \left[X_t' Q X_t + X_t' U i_t + i_t' U X_t + i_t' R i_t\right], \tag{4.3}$$

subject to equation (2.5), where $Q(13 \times 13)$ is a square weighting matrix given by

$$Q = \frac{1}{2}\begin{bmatrix} \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.4}$$

and

$$\underset{(1\times 13)}{U'} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu/2 & 0 \end{bmatrix} \tag{4.5}$$

$$\underset{(1\times 1)}{R} = [\mu/2 + \nu/2]. \tag{4.6}$$

The model is now in a standard linear-quadratic form to which existing optimal control procedures can be used. In appendix B the solution to this problem will be derived in detail. In short, a quadratic solution for the value function is assumed, then a Bellman equation is formed and the FOC derived. A feedback rule for the control variable is then derived and substituted into the right hand side of the Bellman equation. Identification gives the algebraic matrix Riccati equation. A solution, V, to this equation is approached by iterations. The feedback rule is then given by

$$i_t = -\left(R + \delta B'VB\right)^{-1}\left(U' + \delta B'VA\right)X_t \tag{4.7}$$

or

$$i_t = -FX_t, \tag{4.8}$$

where

$$F = \left(R + \delta B'VB\right)^{-1}\left(U' + \delta B'VA\right). \tag{4.9}$$

From equation (4.8) it hence follows that the short term interest rate is set according to the feedback rule F and the present observations on the state vector X_t, which consists of only predetermined variables. As follows from equation (4.9) the feedback rule is independent of the initial value of the state vector $X(0)$ and the covariance matrix of disturbance vector. As a consequence of this, the feedback rule is time consistent, that is, the feedback rule in equation (4.8) is the optimal one regardless if the optimal rule is derived now at time t or in some period $s > t$.

Given the reaction function and the inflation and output equations it is then possible to derive the solution for the equilibrium inflation and output and its unconditional variance. This is done in appendix II. The solutions are

$$\begin{aligned} X_{t+1} &= (A - BF)X_t + \epsilon_{X,t+1} \\ &= MX_t + \epsilon_{X,t+1} \end{aligned} \tag{4.10}$$

and

$$\textstyle\sum_{XX} = M \sum_{XX} M' + \sum_{\epsilon\epsilon}. \tag{4.11}$$

In the sections that follows the numerical solutions using the algorithms above for different specifications of the central banks objective function will be derived and analysed.

4.1. Responses to shocks

As a start the dynamic responses of the variables to different shocks will be analysed. In figures 3-6 it is shown how inflation, output and the interest rate respond to a single one-standard deviation shock to demand and supply, for different values of λ and ν.[11]

In figure 3 the dynamic responses to a supply shock are shown for λ equal to 0.5 and zero, when $\nu \approx 0$.[12] In figure 4 the dynamic responses to a price shock are shown for λ equal to 1 and zero, for $\nu = 0.001$, and in figure 5 the corresponding responses are shown for $\nu = 1$.[13] As can be seen in all diagrams both output and the interest rate fluctuate much less when the central bank not only targets the inflation rate. Further, longer time is used to reduce the annual inflation rate to its target after a shock.

If a supply shock hits the economy and the central bank besides targeting the annual inflation rate also has a target for output, $\lambda = 0.5$, then it again raises the nominal and the real interest rate to create an output-gap which depresses inflation, see figure 3, but less drastic than when it only targeted inflation. It therefore takes longer time, almost two years or seven quarters, before the annual inflation rate is back on its target. Putting some weight on the output target then implies that the central bank more slowly returns the inflation rate to its target. The longer perspective gives a smother development in the interest rate, output and the quarterly inflation rate.

If the central bank has a target for the inflation rate and also wants to avoid too large fluctuations in the interest rate, $\nu = 0.001$, and a supply shock hits the economy, it also raises the nominal and the real interest rate less drastic, see figure 4. In this case it takes around one and a half year, five quarters, before the annual inflation rate is back on its target. Putting some weight on the interest rate target, $\nu = 0.001$, thus implies that the central bank instead of setting the repo rate so that the first controllable inflation forecast equals the annual inflation target, sets the repo rate so that the inflation forecast equals the target when the effect of the repo rate on the inflation rate is at its maximum. The longer perspective again gives a smother development in all variables except the annual inflation rate.

[11] $\mu = 0$ all through the analysis.

[12] The reason for not setting ν exact equal to zero is that the program does not converge then.

[13] The standard deviation in the nominal interest rate for the period 1985 to 1997 is 3.70. Given the simple model of the Swedish economy derived here, it is difficult to reach such a low variability in the interest rate. For ν=1 and $\lambda = 0$ the standard deviation of the interest rate is 6.23.

Combining interest rate smoothing with output stabilization, $\nu = 0.001$ and $\lambda = 1$, reduces the variability in the interest rate and output even further. The initial rise in the interest rate is smaller and consequently the initial fall in output. As can be seen the annual inflation rate is now more volatile. The annual inflation rate approaches its target level between the fourth quarter and the seventh quarter, it then rises again and only slowly moves back towards the target.

When even more weight is put on the interest rate target, $\nu = 1$, the high variability in the interest rate after a supply shock is totally gone, see figure 5. The choice of lambda is no longer of any importance and it takes years to return inflation to target.

In figure 6 the dynamic responses to a demand shock, which here is equal to an output shock, are shown for λ equal to 1 and zero, when $\nu = 0.001$. The shock increases inflation and a policy focusing on inflation stabilization and to some extent interest rate stabilization, $\nu = 0.001$ hence responds by increasing the nominal and therefore also the real interest rate. The purpose is to push the output gap down below the trend, to get annual inflation back on the target. A policy that also tries to stabilise output raises the interest rate more sharply to close the output gap faster, and then returns the interest rate to its trend level to avoid creating a large negative output gap. With more weight put on the interest rate target the output gap is closed more slowly.

The conclusion is hence that the development of the different variables after a shock are much smoother if the central bank puts more weight on the output and/or the interest rate targets. The consequence of this is that it takes longer time to return the annual inflation rate to its target. Another conclusion is that interest rate smoothing and output stabilization have similar implications. The exact implications of the different objectives of the central bank in terms of the volatility of the different variables will now be analysed.

4.2. The optimal policy frontier

In this section the theoretical unconditional standard deviation of inflation, output and the interest rate will be derived for different specifications of the central bank's objective function. In table 4.1 the calculated standard deviations are summarized for all variables and in figures 7-9 the standard deviations in the annual inflation rate and output are shown for different values of λ and ν. The standard deviations are calculated taking the square root of equation (4.11) for the unconditional variance. The discount factor δ is set to 0.9999. The discount factor is set to almost one to get as close as possible to the unconditional variances.

Table 4.1: Theoretical unconditional standard deviations.

λ	ν	Quart. infl.	Annual infl.	Output	Int. rate	Int. rate diff.
0	$\approx$0	66.73	1.27	341	3586	5195
1	$\approx$0	3.73	2.73	2.19	10.17	13.67
0	0.001	3.35	2.03	4.51	27.26	36.27
1	0.001	3.74	2.74	2.18	9.76	12.73
0	1	4.16	3.32	2.15	5.52	2.35
1	1	4.22	3.39	2.03	5.5	2.42

From table 4.1 and figure 7 and 8 it follows not surprisingly that, the standard deviation in the annual inflation rate increases when more weight is put on the output goal, while the opposite is true for the quarterly inflation rate, the output gap and the interest rate, regardless of what weight is put on the interest rate target. As can be seen in figure 8 moving from no weight on the output target to some weight substantially reduce its fluctuations, for $\nu \approx 0$. The impact on the variability in output of a rise in the weight put on output then decreases for larger values of λ, the curves become increasingly flat as the weight on output increases, for $\nu \approx 0$ and also for $\nu = 0.001$. The increase in the variability in the annual inflation rate when more weight is put on the output goals is, in comparison, quite moderate.

A larger standard deviation in the annual inflation rate and a smaller standard deviation in the other variables is also the result if more weight is put on the interest rate goal. This means that the variability in output is substantially reduced without any weight being put on the output goal. If the central bank only cares about inflation the standard deviation in the annual inflation rate is 1.27 and in output it is 341. For $\lambda = 1$ the standard deviation in inflation rises to 2.73 while it is reduced to 2.19 for output. For $\nu = 1$ and $\lambda = 0$ the corresponding figures are 3.32 and 2.15. The weight put on the interest rate target hence has a large impact on the variability in output. As a consequence of this the weight put on the output target is more important for the outcomes of the standard deviations the less the central bank tries to stabilise the interest rate. When $\nu = 1$, the standard deviation for the annual inflation rate is almost unaffected by the weight put on the output target. The decrease in the variability of output and the interest rate when more weight is put on the output goal is also less for higher values of ν. This is illustrated in figures 7 and 8. The curves showing the different standard deviations for output and inflation for $\nu = 1$ are almost horizontal.

The conclusion is hence that when more weight is put on the output stabilization goal and/or on one of the interest rate goals the variability in both the interest rate, output and the quarterly inflation rate are reduced substantially. The cost is higher variability in the annual inflation rate, which is moderate in comparison to the fall in the other variables variability. This is also illustrated in figure 9. There the optimal policy frontier, that is the set of efficient combinations of inflation variance and output variance attainable by policymakers for $\nu = 0.001$ is plotted. As can be seen the curve is quite steep. Further, in figure 10 the optimal policy frontier is drawn for $\nu \approx 0$, $\nu = 0.001$, and $\nu = 1$. As can be seen the frontier is reduced in size as the weight put on the interest rate target is increased. For $\nu = 1$ it looks almost as a point. The picture summarizes what was discussed above, that the weight put on the output target is less important for the outcome of the variability in the annual inflation rate and in output when more weight is put on the interest rate target.

As already mentioned the purpose of this paper is not to evaluate Swedish monetary policy, the structural model used is much too simple. Despite this it can be interesting to compare the calculated standard deviations for the different variables with the actual standard deviations shown in table 4.2.

Table 4.2: Actual standard deviations.

Year	Quart. infl.	Annual infl.	Output	Int. rate	Int. rate diff.
1985-1996	3.74	2.91	2.78	3.70 (2.57)	3.44 (1.13)
1970-1996	4.15	3.27	2.33		

The figures within the parantheses are the standard deviations excluding the extreme observations in the autumn of 1992 when Sweden experienced a crisis in the exchange market and the short term interest rate was rised substantial.

As can be seen the actual standard deviations in output, the quarterly inflation rate and the interest rate are much lower than the standard deviations that the theoretical model with a strict inflation targeting regime produces. The theoretical standard deviation in the annual inflation rate is on the other hand lower than the actual one. It hence seems as if there has not been a strict inflation targeting regime in Sweden. The figures show that the Swedish central bank historically has put a large weight on interest rate smoothing and/or output stabilization.

4.3. Reaction function

When looking at the different reaction functions derived from different specifications of the central bank's objective function, the results are of course in line with those derived above. As already shown in section 3, the coefficients in the reaction function derived when the only object for monetary policy is to keep annual inflation on target, $\lambda = \mu = \nu = 0$, are huge. Written in a slightly different form than in section 3 the reaction function is[14]

$$\begin{aligned} i_t &= 42.1\pi_t + 41.2\pi_{t-1} + 5.4\pi_{t-2} - 7.6\pi_{t-3} + \\ &\quad 4\left(\pi_{t-4} + \pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8}\right) + 2.3\pi_{t-9} + 17.3y_t. \end{aligned} \tag{4.12}$$

The inflation coefficients sum to 103.4 while the output coefficient is 17.3. The large coefficients result from the facts that inflation is brought back to its target very fast after a shock and that the effect of monetary policy after only half a year is small. If the central bank also stabilises output, with $\lambda = 1$, the reaction function is

$$\begin{aligned} i_t &= 2.5\pi_t + 1.6\pi_{t-1} + \pi_{t-2} + 0.3\pi_{t-3} + \\ &\quad 0.8\pi_{t-4} + 0.7\pi_{t-5} + 0.6\pi_{t-6} + 0.4\pi_{t-7} + 0.3\pi_{t-8} + 0.1\pi_{t-9} + 7.7y_t. \end{aligned} \tag{4.13}$$

The weights put on inflation then decrease substantially. When output is also taken into account, the inflation coefficients sum to 8.3. The weight on output also falls, it is now 7.7. The dynamic response in the interest rate when a shock hits the economy is then much smoother, as was shown above. If the quarterly inflation

[14] The reaction function in section 3 was written as

$$\begin{aligned} i_t &= -102.6\pi^* + 42.23\pi_t + 41.3\pi_{t-1} + 5.4\pi_{t-2} - 9.4\pi_{t-3} \\ &\quad +10.3\widetilde{\pi}_{t-3} + 13.7\widetilde{\pi}_{t-4} + 17.2y_t \end{aligned}$$

If the inflation target is set equal to zero and the expression is rewritten they coincide.

$$\begin{aligned} i_t &= 42.23\pi_t + 41.3\pi_{t-1} + 5.4\pi_{t-2} - 9.4\pi_{t-3} \\ &\quad +1.7\left(\pi_{t-3} + \pi_{t-4} + \pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8}\right) \\ &\quad +2.3\left(\pi_{t-4} + \pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8} + \pi_{t-9}\right) + 17.2y_t \\ &= 42.23\pi_t + 41.3\pi_{t-1} + 5.4\pi_{t-2} - 9.4\pi_{t-3} \\ &\quad +4\left(\pi_{t-4} + \pi_{t-5} + \pi_{t-6} + \pi_{t-7} + \pi_{t-8}\right) + 2.3\pi_{t-9} + 17.2y_t \end{aligned}$$

rate today rises by one percentage point (at an annual rate), and assuming for example that the output gap today is closed and the inflation rate $t-1$ and earlier has been on the target then the interest rate should be increased by 2.5 percentage points. The corresponding figure using the reaction function for $\lambda = 0$ is 42.1.

A less volatile interest rate is, as analysed earlier, also achieved if some weight is put on one of the interest rate targets. Further, when the central bank puts some weight on interest rate stabilization, the difference between the outcome when the central bank stabilises output and not is rather small. The reaction functions derived from the two different objective functions, where $\lambda = 0$ and $\lambda = 1$, respectively, when $\nu = 1$ are

$$\begin{aligned} i_t &= 0.7\pi_t + 0.5\pi_{t-1} + 0.3\pi_{t-2} + 0.1\pi_{t-3} + \\ &\quad 0.2\left(\pi_{t-4} + \pi_{t-5}\right) + 0.1\left(\pi_{t-6} + \pi_{t-7} + \pi_{t-8}\right) + 1.2y_t \end{aligned} \tag{4.14}$$

and

$$\begin{aligned} i_t &= 0.7\pi_t + 0.4\pi_{t-1} + 0.3\pi_{t-2} + 0.1\pi_{t-3} + \\ &\quad 0.2\left(\pi_{t-4} + \pi_{t-5} + \pi_{t-6}\right) + 0.1\pi_{t-7} + 0.1\pi_{t-8} + 1.4y_t. \end{aligned} \tag{4.15}$$

The coefficients on inflation and output sum in the first case to 2.3 and 1.2, in the second case they sum 2.3 to and 1.4. The coefficients have hence decreased both for $\lambda = 0$ and $\lambda = 1$, compared with the coefficients derived for a lower weight on the interest rate target. Further the coefficient on output when output is not stabilized, $\lambda = 0$, has decreased more. Table 4.3 summarises the results.

Table 4.3: Weights in the reaction function

λ	μ	$\sum$inflation coefficient	$\sum$output coefficients
0	≈ 0	103.4	17.3
1		8.32	7.7
0	0.001	20.6	6.7
1		8	7.3
0	1	2.3	1.2
1		2.3	1.4

The form of the reaction functions derived above is similar to the Taylor rule (1993), with the difference that Taylor only included output and the inflation rate today in the reaction function. Here also several lags of the quarterly inflation rate are included. Despite this it is interesting to compare the different optimally derived weights for inflation and output in the reaction function above with the weights suggested by Taylor. According to the Taylor rule the weights in the reaction function on inflation should be one and a half and on output one half. As can be seen above even when some weight is put on the interest rate target it is difficult to reach these weights.

As analysed above the results are, besides being a result of the central banks objectives, of course also dependent on the assumed structure of the economy. The background for Taylor suggesting such low weights might have been that he assumed that monetary policy has a much larger impact on output than what the simple macro economic model presented here suggests.

4.4. Quarterly inflation rate smoothing

As analysed above, one reason for there being large swings in all variables but the annual inflation rate was that the quarterly inflation rate had to be adjusted so that the annual inflation target was achieved. The quarterly inflation rate consequently fluctuated heavily. But a high fluctuation in the quarterly inflation rate is probably not something that a central bank caring about inflation would like to experience. Even though central banks usually explicitly only target the annual inflation rate, it is possible that they implicitly also have some target for the quarterly inflation rate. One possibility is that they are interested in smoothing the quarterly changes in the inflation rate, $(\pi_t - \pi_{t-1})$. To analyse the implications of such a target the central bank's objective function is now rewritten to include a target for the quarterly inflation rate. Assuming that the relative weight put on this goal is γ the central bank's loss function in period t is

$$L(X_t) = \frac{1}{2}\left[(\pi_t^y)^2 + \gamma\,(\pi_t - \pi_{t-1})^2 + \lambda y_t^2 + \mu(i_t - i_{t-1})^2 + \nu {i_t}^2\right]. \qquad (4.16)$$

The Q matrix now looks as follows

$$Q = \frac{1}{2}\begin{bmatrix}
\frac{1}{16}+\gamma & \frac{1}{16}-\gamma & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{16}-\gamma & \frac{1}{16}+\gamma & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{16} & \frac{1}{16} & \frac{1}{16} & \frac{1}{16} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \tag{4.17}$$

The theoretical unconditional standard deviations were then derived for different weights on the goals in the objective function. Not surprisingly it turned out that if $\lambda = 1$ or $\lambda = 0$ and $\nu = 0.001$ or $\nu = 1$ the standard deviations were almost the same for different values of γ, including a zero weight as in the previous analysis. The fluctuations in the quarterly inflation rate in those cases were already low, putting some weight on γ therefore did not affect the results.

For the case of strict inflation targeting ($\lambda = 0$ and $\nu \approx 0$) on the other hand, the standard deviations are reduced substantially for all variables except of course for the annual inflation rate when some weight was put on the quarterly inflation smoothing goal. But the standard deviations in output and the short term interest rate are nevertheless quite high. This is shown in table 4.1. The standard deviation in the annual inflation rate is only marginally increased.

Table 4.4: Theoretical unconditional standard deviations

γ	Quart. infl.	Annual infl.	Output	Int. rate	Int. rate diff.
0	66.73	1.27	341	3586	5195
0.5	3.24	1.87	6.8	59.52	90.33
1	3.23	1.98	5.97	53.52	85.57

5. Rational expectations

The Phillips curve derived for Sweden and used in the analysis above was a standard backward looking Phillips curve. The own lags in the inflation equation summed to one. One interpretation of this is that people form their expectations on recent observations and/or that prices are sticky. An alternative approach is to use an expectations augmented Phillips curve. It could for example be of the following form

$$\pi_{t+1} = (1 - \delta_\pi)\, \pi_{t+2|t} + \delta_\pi \pi_t + \alpha_y y_t + \epsilon_{t+1} \tag{5.1}$$

where the coefficients on the inflation expectation term and the own lags are assumed to sum to one and δ_π captures the stickiness of inflation. If the central bank only targets inflation, and its policy is credible, then the expected future inflation is given by the target

$$\pi_{t+2|t} = \pi^* \tag{5.2}$$

The Phillips curve can then be written as

$$\pi_{t+1} = (1 - \delta_\pi)\, \pi^* + \alpha_\pi \pi_t + \alpha_y y_t + \epsilon_{t+1} \tag{5.3}$$

With no stickiness, $\delta_\pi = 0$, the Phillips curve is

$$\pi_{t+1} = \pi^* + \alpha_y y_t + \epsilon_{t+1} \tag{5.4}$$

With complete stickiness, $\delta_\pi = 1$, the Phillips curve is

$$\pi_{t+1} = \pi_t + \alpha_y y_t + \epsilon_{t+1} \tag{5.5}$$

The coefficient on lagged inflation is hence unity in the last example. This implies that if the estimated inflation equation used in this paper is a reduced form of an expectations augmented Phillips curve then the results would imply that prices are complete sticky, assuming that the only target for the central bank was the inflation rate.

The question is then how the results in this paper would be effected if an expectations augmented Phillips curve, where $0 \leq \delta_\pi < 1$, were used instead of a backward looking Phillips curve. It follows from above that the persistence in inflation would fall. An implication of this is that the optimal policy frontier moves towards the origin. It then follows that monetary policy can be less aggressive under a strict inflation targeting regime than showed here. The cost of achieving the inflation target in terms of the variability in output can hence be reduced. In the extreme case with no inflation stickiness, $\delta_{\pi} = 0$, it is hence possible for the central bank to achieve the inflation target with no cost.[15]The extreme results derived here for only an inflation target would hence probably disappear. For the cases when the central bank has additional goals the results would also be modified toward less volatility in the different variables, but probably not as much as when only inflation is targeted.

What is then the motivation for not using an expectations augmented Phillips curve in this paper? Theoretically it is straight forward to include expectations in the model. Empirically, this is associated with quite a lot of problems, though. The easiest way to include expectations in the model empirically is to use inflation expectations data from surveys. The problem for the Swedish case in this aspect is that there are no long series of inflation expectations. There is one time series over households inflation expectations that starts in 1979.[16] Further, to use survey data tests have to be made to ensure that the expectations captured in the survey are rational. Roberts (1996) shows that the Michigan survey of inflation expectations did not make econometrically efficient use of available information. He further refers to a lot of other studies that also shows that inflation expectations captured in surveys are not perfectly rational. If on the other hand expectations are estimated, problems might arise if there has been monetary policy regime shifts during the estimated period. To use this method it is hence necessary to try to identify such shifts and try to control for them. What is a priori known is that the ultimate objective for Swedish monetary policy has been price stability for many years. An explicit target for the inflation rate was not introduced until 1995 however. What the target was for inflation before that is unclear. Further, it is unclear whether the Swedish central bank besides its concern for the inflation rate also tried to stabilise for example output.

To summarize, to be able to estimate an expectations augmented Phillips curve some econometrically complicated issues have to be solved. Since the main purpose of this paper is not to estimate a complete macro economic model for the Swedish economy this issue is not dealt with here. In a future paper these questions will be addressed. The purpose of this paper is hence only to take a first step in characterizing monetary policy in Sweden. For that purpose only a very simple macro economic model is used.

[15]This result follows from the New Keynesian models with sticky prices, since those models do not predict that inflation will be sticky.

[16]In "Hushållens Inköpsplaner", the Consumer Buying Expectations report, published by Statistics Sweden.

6. Conclusions

Using a simple macroeconomic model of the Swedish economy and a model of optimal monetary policy it has been illustrated how the central bank's stabilization goals affect the perspective of the central bank, the determination of the short term interest rate and the variability of inflation, output and the interest rate.

In a strict inflation targeting regime, where the central bank only has a goal for the annual inflation rate, it focuses on the shortest controllable horizon. In this model it is the forecast of the annual inflation rate half a year ahead. This is hence a very short perspective and the consequence of this is that the short term interest rate has to be changed dramatically when deviations occur between the inflation forecast and the inflation target. The fluctuations in the annual inflation rate in this regime will be quite small while the fluctuations in the quarterly inflation rate, output and the short term interest rate on the other hand will be huge.

When the objective function includes additional stabilization goals the results are changed quite drastically. With some weight put on the output stabilization goal, (here in this analysis a weight of 0.5) almost two years are used to adjust the inflation rate back toward its target after a shock. This less aggressive monetary policy results in a dramatic fall in the variability in the quarterly inflation rate, output and the short term interest rate. The cost is a moderate increase in the variability in the annual inflation rate.

Similar results are reached if more weight is put on the interest rate goal. With a weight of 0.001 on this goal, in this analysis, one and a half year is used to bring inflation back to its target after a shock, which is when the repo rate has its largest effect on the inflation rate.

A strict inflation targeting regime that focuses on the annual inflation rate turns out to have large implications for the quarterly inflation rate. As in the cases of output and the short term interest rate the quarterly inflation rate fluctuates heavily. Assuming that such an outcome is not desirable suggests an inclusion of a separate quarterly inflation rate smoothing goal in the objective function. The variability in the quarterly inflation rate is then substantially reduced. Also the variability in output and the short term interest rate is reduced, although they still are high compared with the outcomes when some weight also is put on one of these goals.

Finally, the theoretical standard deviations were compared with the actual standard deviations in Swedish data 1985-1996. The results indicate, bearing the simplicity of the macro economic model in mind, that the Swedish central bank has put a lot of weight on interest rate smoothing and/or on output stabilization.

Table A.1: Augmented Dickey Fuller Test.

Variable	ADF Test statistic	Critical Value	LM-test[17], F-st, P-val	n[18]
$i - \pi_t^y$	-3.24	5% -2.9	0.98 [0.38]	1
π	-2.25	10% -2.58	1.89[0.16]	3
y	-2.81	10% -2.58	1.45 [0.24]	5

$$\Delta X_t = c + \beta_0 X_{t-1} + \sum_{i=1}^{n} \beta_i \Delta X_{t-i} + \epsilon_t$$

A. Empirical results

First a unit root test, the Augmented Dicky Fuller test, was made to examine the stationarity of the different time series. The results are presented in table A.1.

On a 5 % significance level the hypothesis that the real interest rate contains a unit root can be rejected. On a 10 % significance level the output gap is also stationary. On a 10 % significance level the hypothesis that the quarterly inflation rate contains a unit root can not be rejected.

Since the series that will be used when estimating the output equation are stationary it is meaningful to apply standard inference procedures to this regression. The following equation was estimated with the OLS-method,

$$y_{t+1} = 0.9y_t - 0.13(i_t - \frac{1}{4}(\pi_t - \pi_{t-1} - \pi_{t-2} - \pi_{t-3})) + \eta_{t+1}. \tag{A.1}$$

The output equation is estimated on quarterly data from the end of 1985 to 1996. The reason for not using earlier data is that the Swedish financial markets were highly regulated until 1985. Monetary policy was before 1985 conducted by changes in the regulation of banks and other financial institutions. Since then the instrument of monetary policy has been the short term interest rate. The inflation equation on the other hand is estimated using data from 1970. Since the available number of observations was different for the two equations the equations were estimated separately.

As discussed in section 2 the inflation equation was first estimated in the following form

$$\pi_{t+1} = 0.34\pi_t + 0.18\pi_{t-1} + 0.36\pi_{t-2} - 0.28\pi_{t-3} + 0.4\widetilde{\pi}_{t-4} + 0.3y_t + \epsilon_{t+1}. \tag{A.2}$$

The reason for using the variable $\widetilde{\pi}_t = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3} + \pi_{t-4} + \pi_{t-5})/6$ instead of using the variables separately is that the exact parameter values of lags

Table A.2: Estimation results for the inflation and output equations.

	Inflation equation, π_t	Inflation equation, $\Delta\pi_t$	Output equation
Constant	-0.15 (-0.47)	-0.14 (-0.45)	0.03 (0.22)
Lagged inflation, $t-1$	0.33* (3.24)	-0.67* (-6.66)	
$t-2$	0.17** (1.77)	-0.48* (-4.10)	
$t-3$	0.36* (3.62)	-0.12 (-0.93)	
$t-4$	-0.28* (-2.66)		
$(t-5+..+t-10)/6$	0.33* (2.24)	-0.4* (-3.32)	
$\sum$	0.91		
Outputgap, $t-1$	0.30* (2.11)	0.30* (2.19)	0.91* (21.16)
Real int.rate, $t-1$			-0.13* (-3.94)
R^2	0.51	0.45	0.93
S.E. of regression	3.05		0.78
Ljung-Box Q-st.[19]	0.44 [0.998]	0.71 [0.99]	9.26 [0.16]
LM-test, F-st.[20]	0.58 [0.68]	0.69 [0.6]	1.81 [0.15]
White's F-st	1.15 [0.33]	1.26 [0.27]	0.78 [0.54]
Jarque-Bera	[0.01]	[0.04]	[0.48]

Figures within parentheses are t-statistics, figures within brackets are p-values.
* Significantly different from zero at the 5 percent level. ** Significantly different from zero at the 10 percent level.

further back is of little interest. By making an average measure it is also possible to reduce the parameter space. A redundant variable test was also made for this variable and the result was that it should be included in the estimation.

Given that the inflation process, using data from this period, was characterized by a unit root, the inflation equation was estimated in the following standard form

$$\Delta\pi_{t+1} = \alpha'_{\pi}(L)\Delta\pi_t + \alpha_y(L)y_t + \epsilon_{t+1}, \tag{A.3}$$

from which equation (2.3) was derived. Table A.2 summarizes the results.

As can be seen the different diagnostic tests reveal no evidence of poor specification apart from non-normality of the residuals in the inflation equations, resulting from extreme values. The Ljung-Box Q-statistic that test the hypothesis that the first six autocorrelations are zero is not rejected. The LM-test, also testing for the existence of autocorrelation, show no evidence of serial correlation. The White's Heteroskedasticity test of whether the errors are homoskedastic is not rejected. The Jarque-Bera statistic, testing for normality, show sign of non-normality of the residuals from the inflation equations regressions.

B. The central bank's optimization problem

In matrix notation the optimization problem is

$$J(X_t) \equiv \min_{\{i_t\}_{t=1}^{\infty}} \mathrm{E}_t \sum_{t=1}^{\infty} \delta^{\tau-t} \left[X_t'QX_t + X_t'Ui_t + i_t'U'X_t + i_t'Ri_t\right], \qquad \text{(B.1)}$$

subject to

$$X_{t+1} = AX_t + Bi_t + \epsilon_{X,t+1}. \qquad \text{(B.2)}$$

The model is in a standard linear-quadratic form to which existing optimal control procedures can be used.[21] One can then assume that the value function is quadratic.

$$J(X_t) = \left(X_t'V_tX_t + W_t\right), \qquad \text{(B.3)}$$

where V_t is the value function at time t which is the discounted value of expected losses evaluated along the optimal program. The Bellman equation to this problem hence becomes

$$X_t'V_tX_t + W_t \equiv \min_{i_t} \left\{ \begin{array}{c} X_t'QX_t + X_t'Ui_t + i_t'U'X_t + i_t'Ri_t \\ +\delta \mathrm{E}_t \left(X_{t+1}'V_{t+1}X_{t+1} + W_{t+1}\right) \end{array} \right\}. \qquad \text{(B.4)}$$

The first-order necessary condition for the problem is;

$$\begin{aligned} \frac{\partial J(X_t)}{\partial i_t} &= X_t'U + U'X_t + 2Ri_t + \delta 2B'V_{t+1}\left(AX_t + Bi_t\right) \qquad \text{(B.5)} \\ &= U'X_t + Ri_t + \delta B'V_{t+1}AX_t + \delta B'V_{t+1}Bi_t \\ &= \left(U' + \delta B'V_{t+1}A\right)X_t + \left(R + \delta B'V_{t+1}B\right)i_t = 0. \end{aligned}$$

From the first order condition it is then possible to derive a reaction function for the short term interest rate equal to

$$i_t = -\left(R + \delta B'V_{t+1}B\right)^{-1}\left(U' + \delta B'V_{t+1}A\right)X_t \qquad \text{(B.6)}$$

or

$$i_t = -F_tX_t, \qquad \text{(B.7)}$$

[21] In Chow (1975) and in Sargent (1987) a solution to this kind of a problem is presented. Their solutions are used here.

where

$$F_t = \left(R + \delta B' V_{t+1} B\right)^{-1} \left(U' + \delta B' V_{t+1} A\right). \tag{B.8}$$

To solve for V_t and W_t, the interest rate i_t is substituted into equation B.4.

$$\begin{aligned}
\left(X_t' V_t X_t + W_t\right) &\equiv X_t' Q X_t + X_t' U \left(-F_t X_t\right) + \left(-F_t X_t\right)' U' X_t + && \text{(B.9)} \\
&\quad \left(-F_t X_t\right)' R \left(-F_t X_t\right) + \delta \mathrm{E}_t \left(X_{t+1}' V_{t+1} X_{t+1} + W_{t+1}\right) \\
&= X_t' Q X_t - X_t' U F_t X_t - X_t' F_t' U' X_t + X_t' F_t' R F_t X_t + \\
&\quad \delta \mathrm{E}_t \left(X_{t+1}' V_{t+1} X_{t+1} + W_{t+1}\right) \\
&= X_t' \left(Q - U F_t - F_t' U' + F_t' R F_t\right) X_t + \delta \mathrm{E}_t X_{t+1}' V_{t+1} X_{t+1} \\
&\quad + \delta \mathrm{E}_t W_{t+1} && \text{(B.10)} \\
&= X_t' \left(Q - U F_t - F_t' U' + F_t' R F_t\right) X_t + \\
&\quad \delta \left(A X_t - B F_t X_t\right)' V_{t+1} \left(A X_t - B F_t X_t\right) + \delta \mathrm{trace}\left(V_{t+1} \textstyle\sum_{XX}\right) \\
&\quad + \delta \mathrm{E}_t W_{t+1} && \text{(B.11)} \\
&= X_t' \left(Q - U F_t - F_t' U' + F_t' R F_t\right) X_t + \\
&\quad \delta X_t' \left(A - B F_t\right)' V_{t+1} \left(A - B F_t\right) X_t + \delta \mathrm{trace}\left(V_{t+1} \textstyle\sum_{XX}\right) \\
&\quad + \delta \mathrm{E}_t W_{t+1} && \text{(B.12)} \\
&= X_t' \left(Q - U F_t - F_t' U' + F_t' R F_t + \delta \left(A - B F_t\right)' V_{t+1} \left(A - B F_t\right)\right) X_t \\
&\quad + \delta \mathrm{trace}\left(V_{t+1} \textstyle\sum_{XX}\right) + \delta \mathrm{E}_t W_{t+1}
\end{aligned}$$

using the solution for the state variable which is

$$\begin{aligned}
X_{t+1} &= \left(A - B F_t\right) X_t + \epsilon_{X,t+1} && \text{(B.13)} \\
&= M_t X_t + \epsilon_{X,t+1}.
\end{aligned}$$

Identification gives that

$$\begin{aligned}
V_t &= Q - U F_t - F_t' U' + F_t' R F_t + && \text{(B.14)} \\
&\quad \delta \left(A - B F_t\right)' V_{t+1} \left(A - B F_t\right) && \text{(B.15)} \\
W_t &= \delta \mathrm{trace}\left(V_{t+1} \textstyle\sum_{\epsilon\epsilon}\right) + \delta W_{t+1}, && \text{(B.16)}
\end{aligned}$$

where $\sum_{\epsilon\epsilon}$ is the conditional variance-covariance matrix of ϵ_t in X_t

$$\textstyle\sum_{\epsilon\epsilon} = \mathrm{E}\left(\epsilon_{Xt} \epsilon_{Xt}'\right). \tag{B.17}$$

A solution to equation B.14 and consequently B.7 can be approached by iterations, starting from $V_{t+1} = 0$.

Next the theoretical unconditional covariance matrix of X_t will be derived. It is

$$\begin{aligned}\sum_{XX} &= \mathrm{E}\left(X_t X_t'\right) = \mathrm{E}\left[\left(MX_{t-1} + \epsilon_{X,t}\right)\left(MX_{t-1} + \epsilon_{X,t}\right)'\right] \\ &= M\mathrm{E}\left(X_{t-1}X_{t-1}'\right)M' + \mathrm{E}\left(\epsilon_{Xt}\epsilon_{Xt}'\right) \\ &= M\sum_{XX} M' + \sum_{\epsilon\epsilon}.\end{aligned} \tag{B.18}$$

A solution to this equation can be obtained in terms of a vec operator. If a vec operator is applied to both sides of the equation the result is

$$\begin{aligned}\mathrm{vec}\left(\sum_{XX}\right) &= \left(M \otimes M\right)\mathrm{vec}\left(\sum_{XX}\right) + \mathrm{vec}\left(\sum_{\epsilon\epsilon}\right) \\ &= N\mathrm{vec}\left(\sum_{XX}\right) + \mathrm{vec}\left(\sum_{\epsilon\epsilon}\right),\end{aligned} \tag{B.19}$$

where $N \equiv M \otimes M$ and is of dimension $(7^2 \times 7^2)$. The solution is

$$\mathrm{vec}\left(\sum_{XX}\right) = \left(I_{7^2} - N\right)^{-1}\mathrm{vec}\left(\sum_{\epsilon\epsilon}\right), \tag{B.20}$$

provided that the matrix $(I_{7^2} - N)$ is nonsingular and hence has an inverse.

C. Graphs

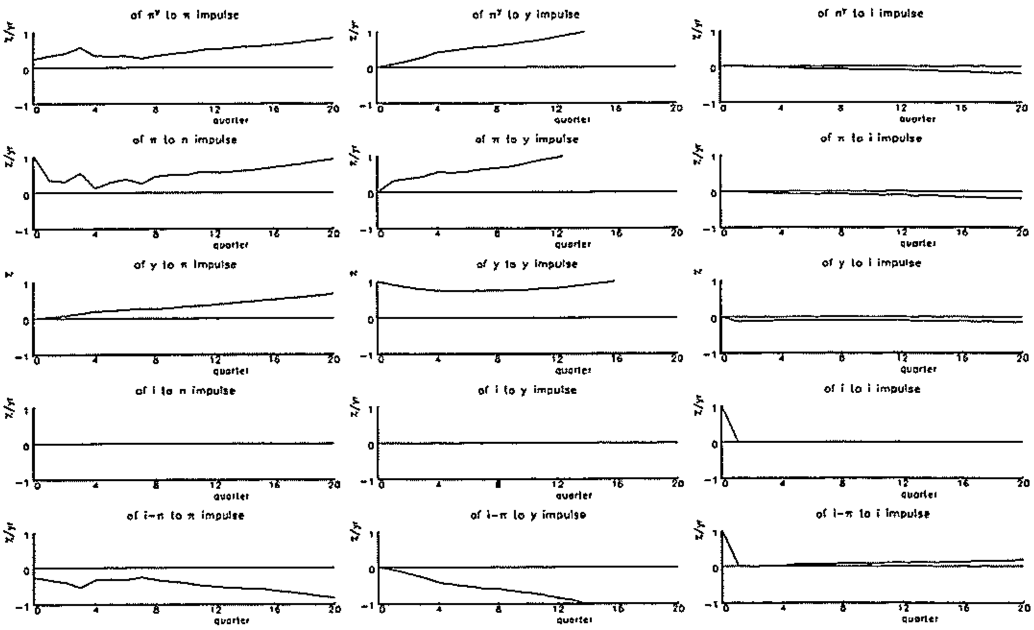

Figure 1: Dynamic responses in the baseline-case.

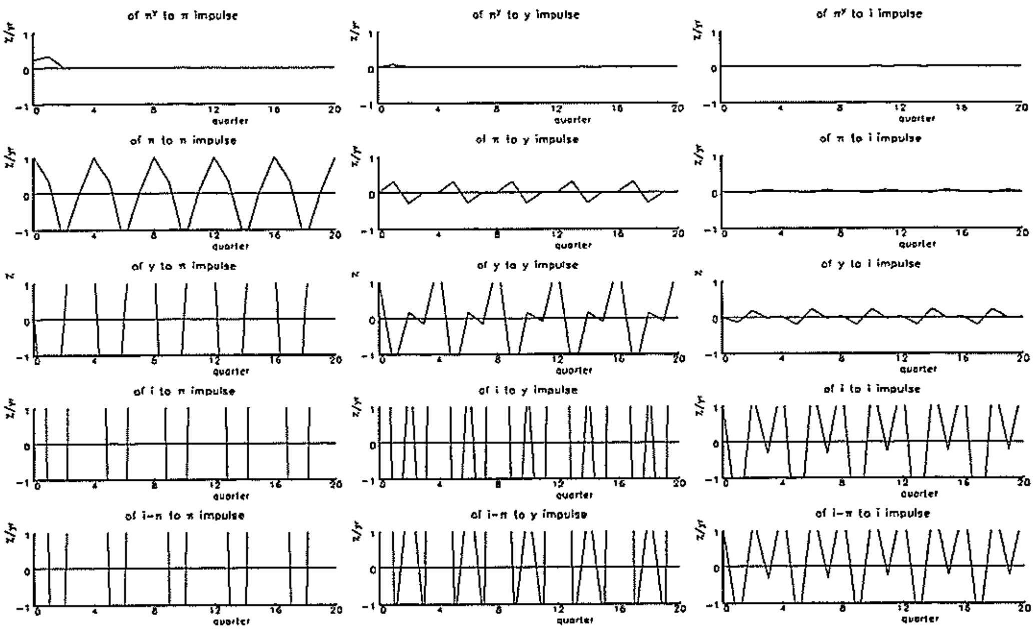

Figure 2: Dynamic responses in a strict inflation targeting regime.

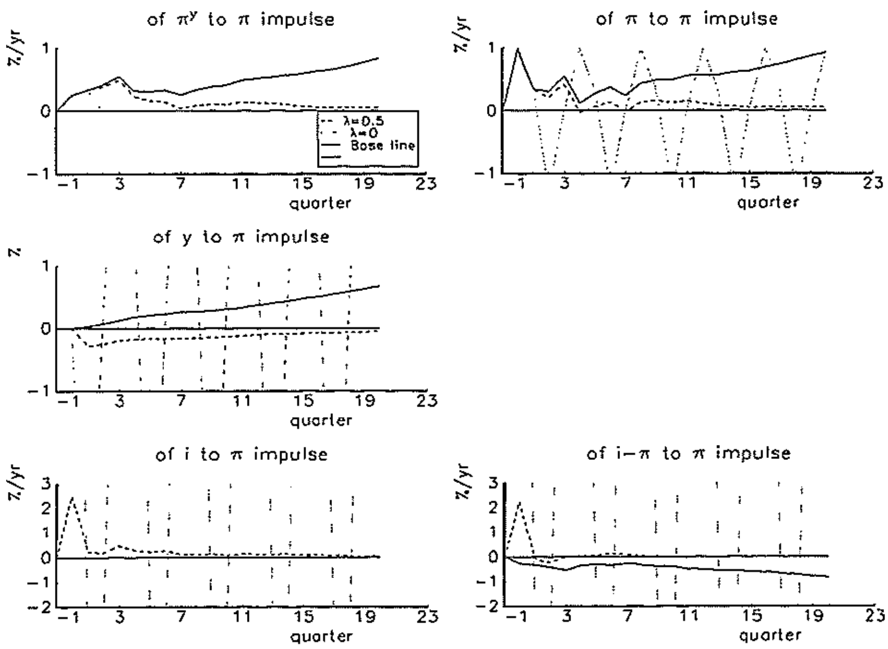

Figure 3: Dynamic responses to a supply shock for λ=0.5 and λ=0 when v=0.

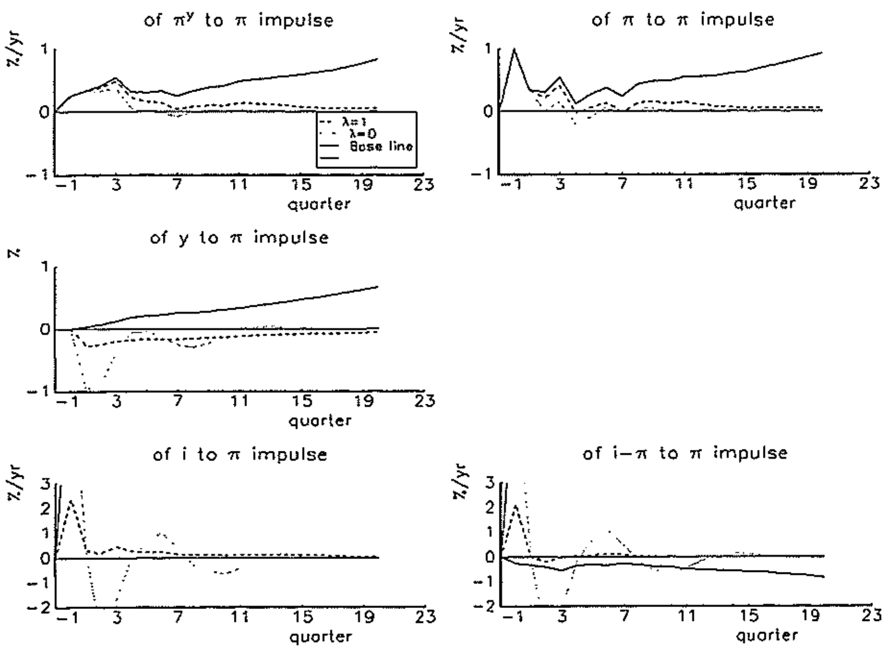

Figure 4: Dynamic responses to a supply shock for λ=1 and λ=0 when v=0.001.

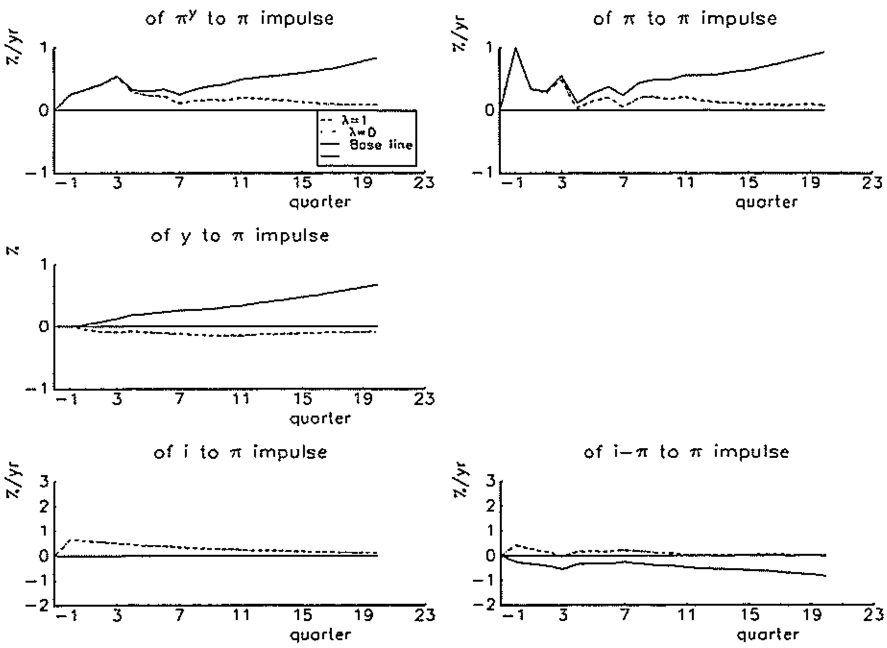

Figure 5: Dynamic responses to a supply shock for λ=1 and λ=0 when v=1.

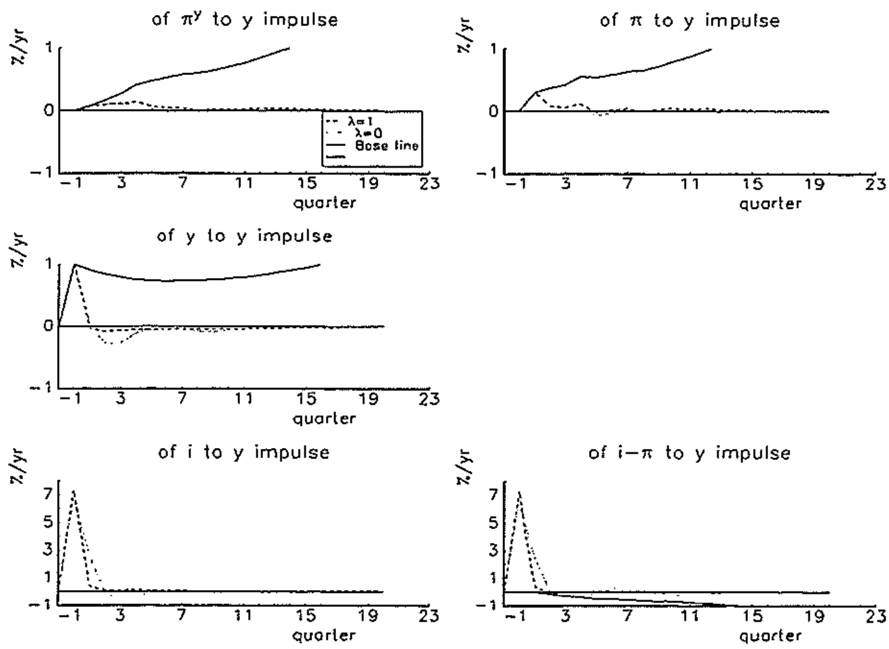

Figure 6: Dynamic responses to a demand shock for λ=1 and λ=0 when v=0.001.

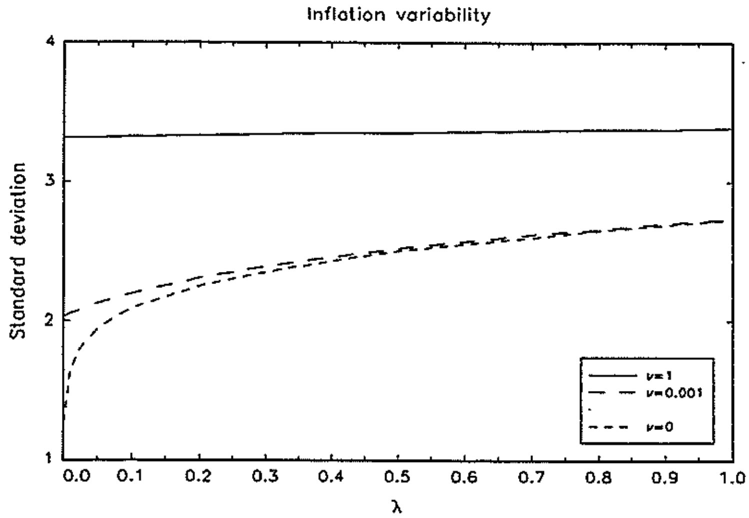

Figure 7.

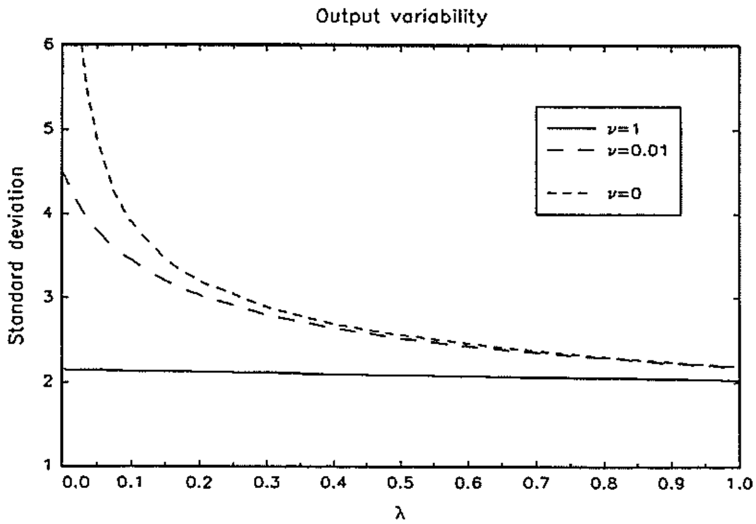

Figure 8.

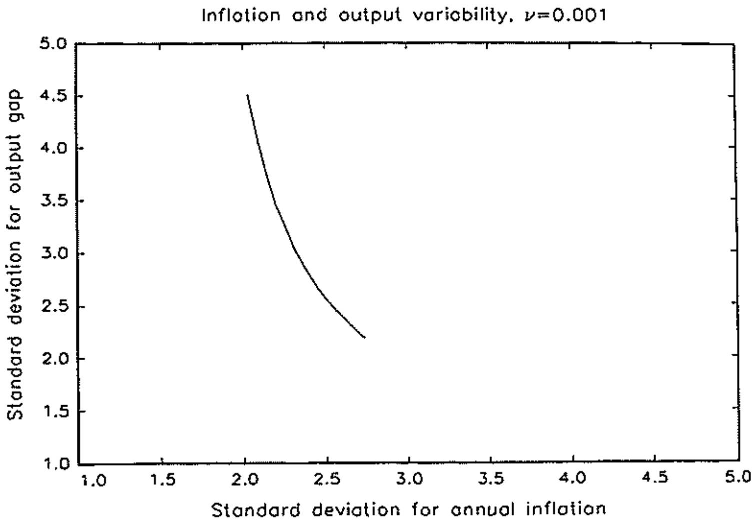

Figure 9.

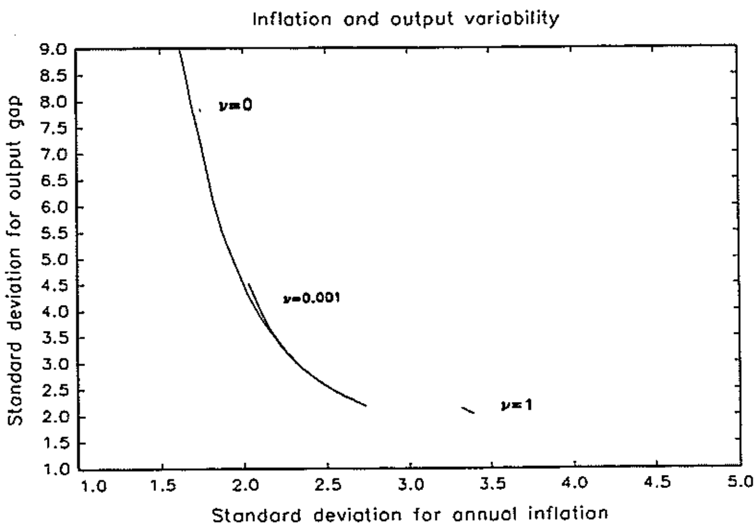

Figure 10.

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